Exercise 56

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$$\sin x = x^2 - x, \quad (1, 2)$$

Solution

Bring all terms to one side.

$$\sin x - x^2 + x = 0, \quad (1, 2)$$

Let $f(x) = \sin x - x^2 + x$. The trigonometric and polynomial functions are each continuous on their respective domains by Theorem 7. The sum or difference of these functions is also continuous by Theorem 4.

$$f(x) = 0, \quad (1, 2)$$

Find a value of x in the interval $[1, 2]$ so that $f(x)$ is negative, and find a value of x in the interval $[1, 2]$ so that $f(x)$ is positive.

$$f(1) \approx 0.841$$

$$f(2) \approx -1.09$$

$f(x)$ is continuous on the closed interval $[1, 2]$, and $N = 0$ lies between $f(1)$ and $f(2)$. By the Intermediate Value Theorem, then, there exists a number c such that $f(c) = 0$.